Prof. Dale van Harlingen, UIUC, Physics 498 Superconducting Quantum Devices

Lecture 2: Defining properties of superconductors

Explore the properties that define the superconducting state:

1. Zero resistance
2. Limits to superconductivity: critical temperature, critical magnetic field, critical current
3. Perfect diamagnetism --- the Meissner effect
4. Limits to the Meissner effect:

    (a) penetration depth

    (b) vortices --- Type II superconductivity

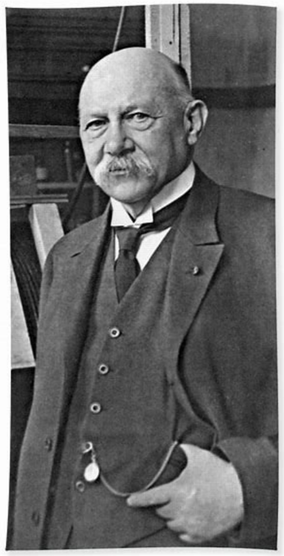
    (c) anisotropy --- pancake vortices

    (d) geometric effects --- Intermediate state

    (e) surface superconductivity

Basic properties of SCs:

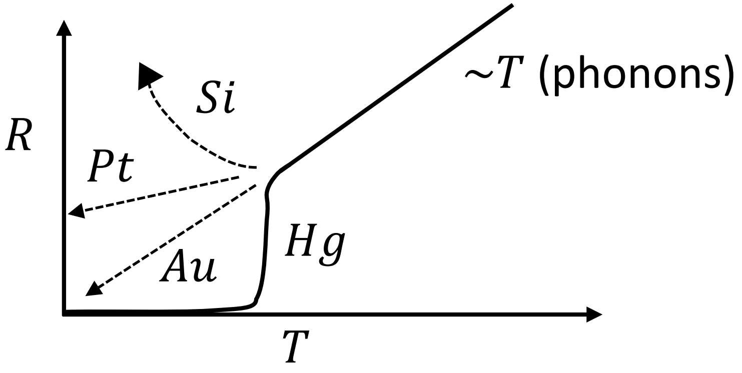
1. ZERO ELECTRICAL RESISTANCE – primary defining property; one of the hardest to explain



* Discovery: Heike Kamerlingh Onnes   Leiden 1911 (April)

First to liquefy helium

Studying normal metal conductance $R\ vs. T$



Si turns up
Pt leveled off
$Au \rightarrow 0$ gradually
$H_g \rightarrow 0$ abruptly

Good experimentalist – 1st observed phenomenon, then tried to the understand behavior.

Discovered:

Au, Pt were not SC
Hg, Pb, Sn were SC

Defined critical temperature
Defined critical magnetic field

Defined critical current

Found that the impurity content was not important (Anderson Theorem 1959)

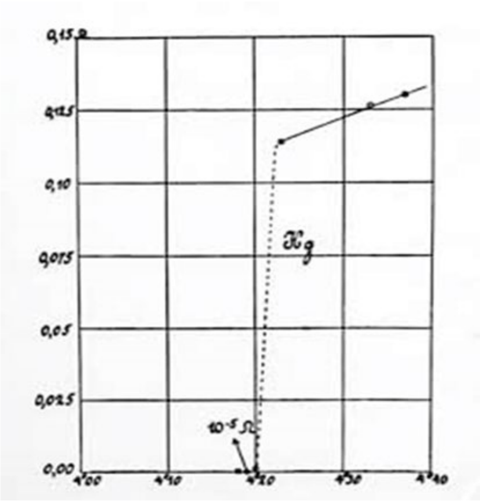
Named it SUPRA-CONDUCTIVITY

* Nobel Prize 1913

History of Discovery:

IEEE Trans. Magnetic 23 (1987) -- 75th Anniversary
Physics Today **63**, 9, 38 (2010) --- 100TH Anniversary

R



Two key questions:

1. How "zero" is the resistance?

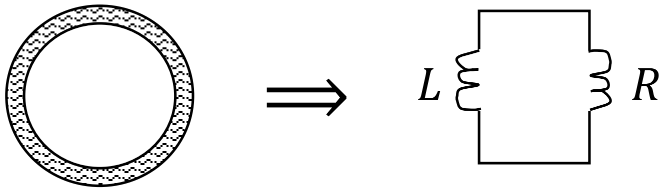
H_g $\quad \rho(300K) \sim 10^{-4}\Omega$–cm

$\rho(0K) < 10^{-12}\ \Omega$–cm $\qquad$ factor of 10^8 lower $\rightarrow$ really small

For comparison, "pure" Cu $\quad \rho(0K) \sim 10^{-10}\Omega$–cm

Advanced experiments to test ρ

SC ring



$$\tau = \frac{L}{R}$$

$$L\,\frac{dI}{dt} + RI = 0 \quad \Rightarrow \quad I(t) = I(0)\, e^{-\left(\frac{R}{L}\right)t}$$

Apply magnetic field to induce a circulating current and monitor the decaying field in the loop for as long as you are willing to wait

Best result: *Pb* $\quad \rho(0K) < 10^{-25}\ \Omega$–cm

$\rho(300K) \sim 10^{-5}\Omega$–cm

factor of 10^{20} lower! $\rightarrow$ It's really zero!

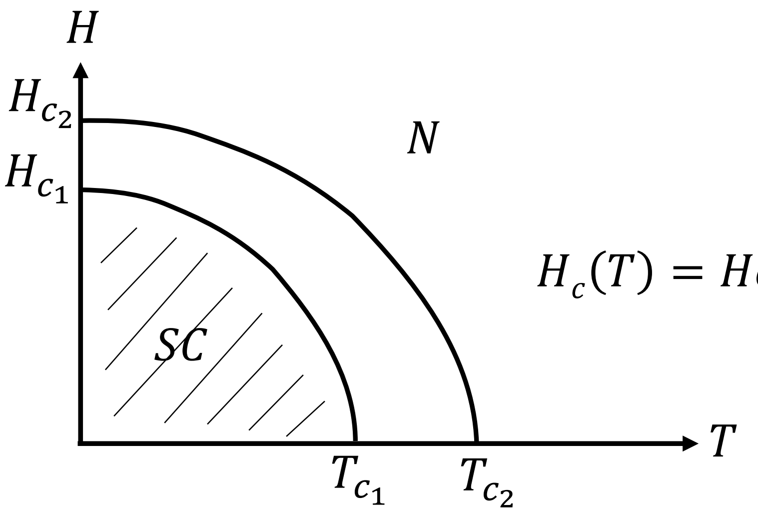
2. How sharp is the transition? --- is it an abrupt or a gradual change

Depends on purity and homogeneity
Broadened by magnetic fields and fluctuations

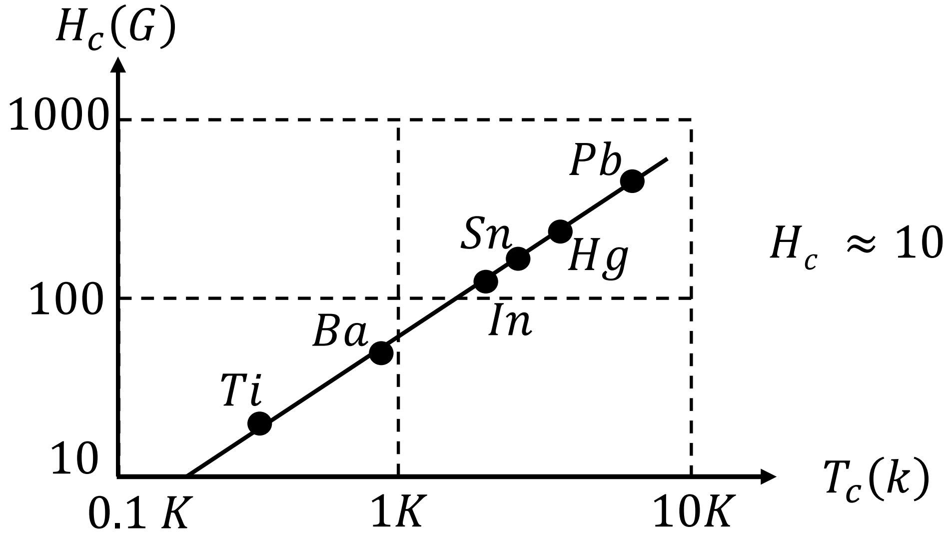
Best results $\Delta T < 10^{-5}\ K$!

2. CRITICAL TEMPERATURE/MAGNETIC FIELD/CURRENT

- SC destroyed by temperature T_c critical temperature
- SC destroyed by magnetic field H_c critical field



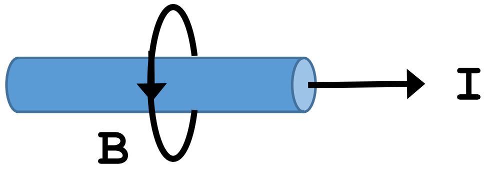
$$H_c(T) = Hc(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$



$H_c \approx 100\ G/K$

- SC destroyed by current flow I_c critical current

"Silsbee rule" - wire becomes normal when field at the surface reaches H_c



$$B = \frac{\mu_0 I}{2\pi r} \Rightarrow \boxed{I_c = 2\pi r\, H_c/\mu_0}$$

[not strictly true --- as we will see, geometry & microscopic effects "pairbreaking" --- play a big role as well]

3. PERFECT DIAMAGNETISM ($\vec{B} = O$) Magnetic field excluded from SC

Discovery : Walther Meissner and Robert Ochsenfeld 1933 "Meissner effect"

Meissner

Ochsenfeld

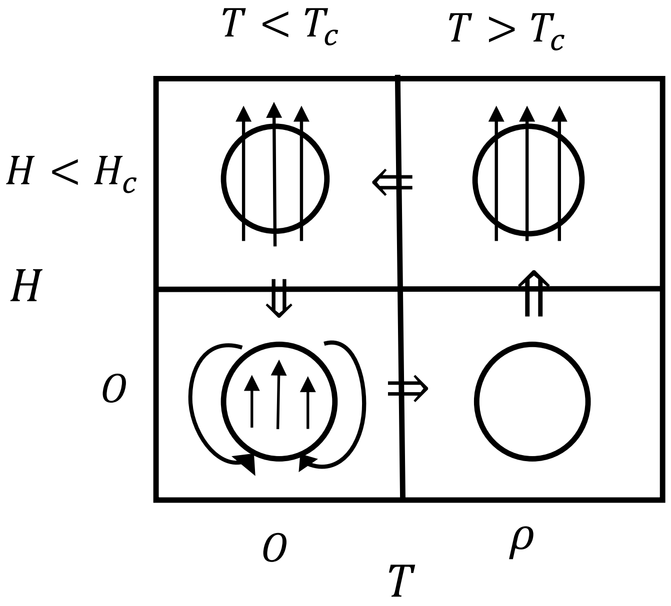
This was a surprise – why?

Consider a perfect conductor $\rho = O \Rightarrow \vec{E} = 0$

Maxwell's equations : $\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt} = O \Longrightarrow \frac{d\vec{B}}{dt} = O$ $\therefore \vec{B}$ = constant

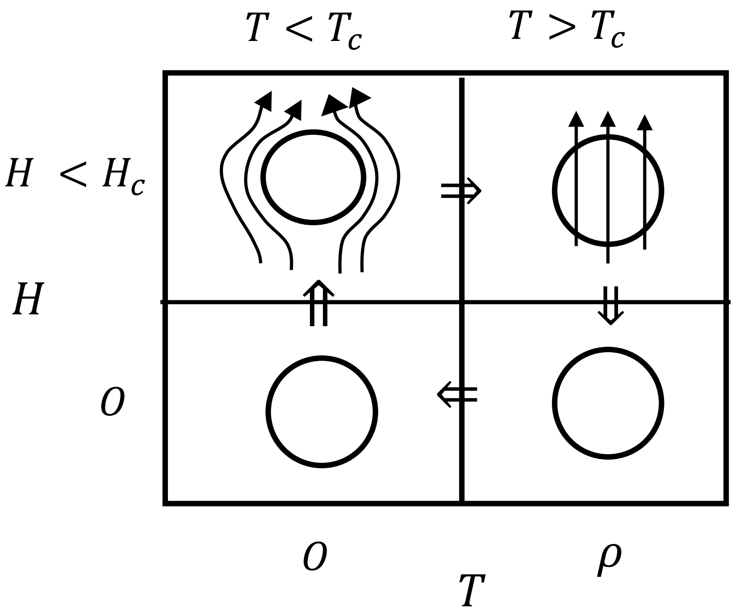
FINITE FIELD-COOLED

field trapped in



ZERO FIELD-COOLED

field excluded



For a perfect conductor, the state below T_c depends on the history of the cooling

For a superconductor, the state below T_c behaves as zero-field cooled always! $\vec{B} = 0$ inside SC (not only $\rho = 0$)

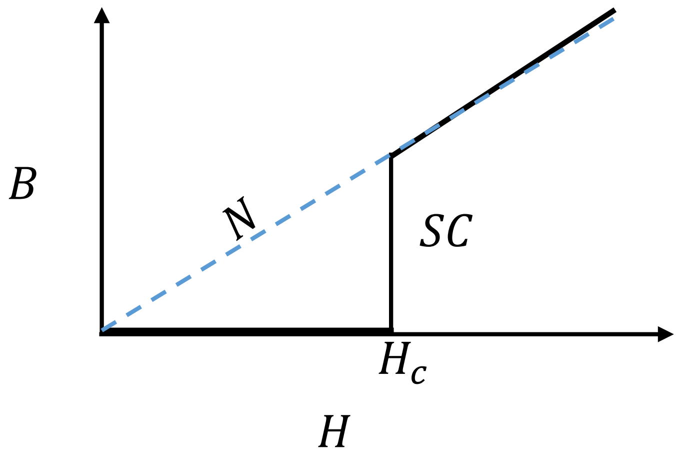
Describe state as a PERFECT DIAMAGNET :

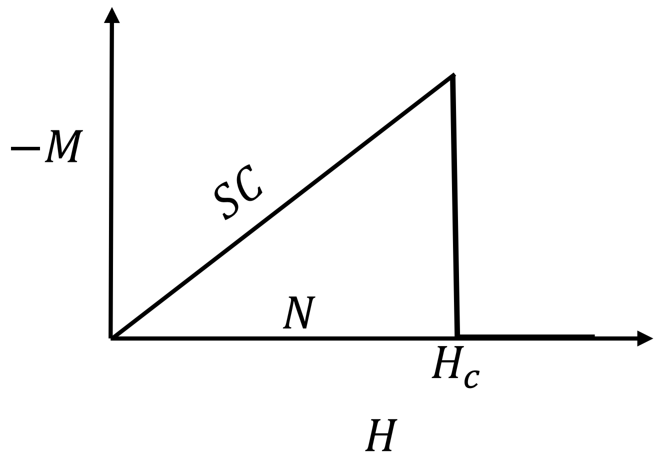
$\vec{B} = \mu_0(\vec{H} + \vec{M})$ [MKS]

$\vec{B} = 0 \rightarrow \vec{M} = -\vec{H}$

$\vec{B} = \vec{H} + 4\pi\vec{M}$ [cgs]

$\vec{B} = 0 \Longrightarrow \vec{M} = -\frac{1}{4\pi}\vec{H}$





Two possible descriptions to give these field behaviors:

(1) perfect diamagnetic material (local) → this picture has some advantages for energy calculations

(2) screening supercurrents flow to screen fields (global) → this is the accurate physics description

4. **MAGNETIC FIELD PENETRATION**

What we have discussed almost never occurs --- fields penetrate superconductors for many reasons:

(a) Penetration depth

Effective diamagnetism caused by screening currents that flow over a finite thickness near edge

“penetration depth” $\lambda = \sqrt{\frac{m}{ne^2\mu_0}}$

Fields penetrate into sample over this distance --- we will characterize this phenomenon in various electrodynamic and microscopic theoretical models

(b) Type II superconductivity

Shubnikov (experiments); Abrikosov (theory)

Most SC are Type II and allow flux penetration in discrete “vortices” of magnetic flux

Type I SC : elements $<$ 1% of all superconductors

Type II SC : alloys, compounds, thin films, dirty materials, disordered, unconventional, ...

*Important – allows SC $(R = 0)$ at fields well above the 100 g/K region

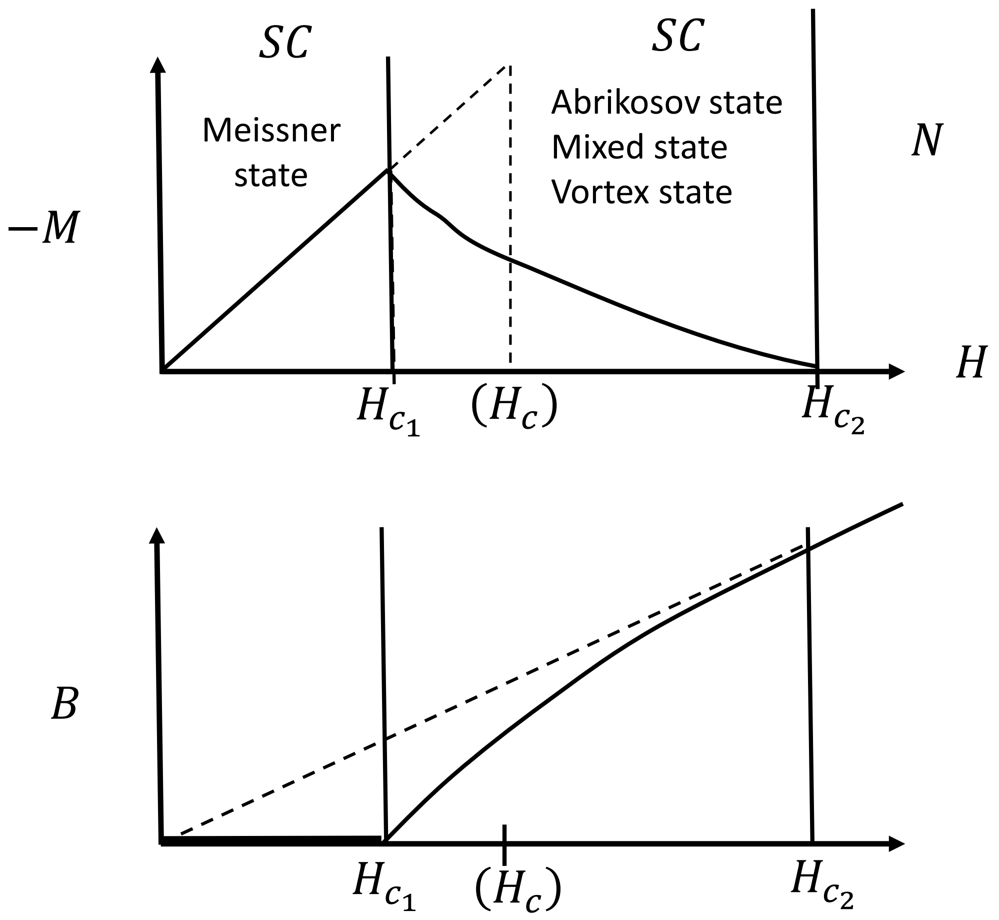
(if NbTi were type I, Hc $\sim$1 KG instead of > 10 T)
$\times 100$

Key : surface energy positive (Type I) or negative (Type II) --- we will study the theory in depth

interface between N and S regions

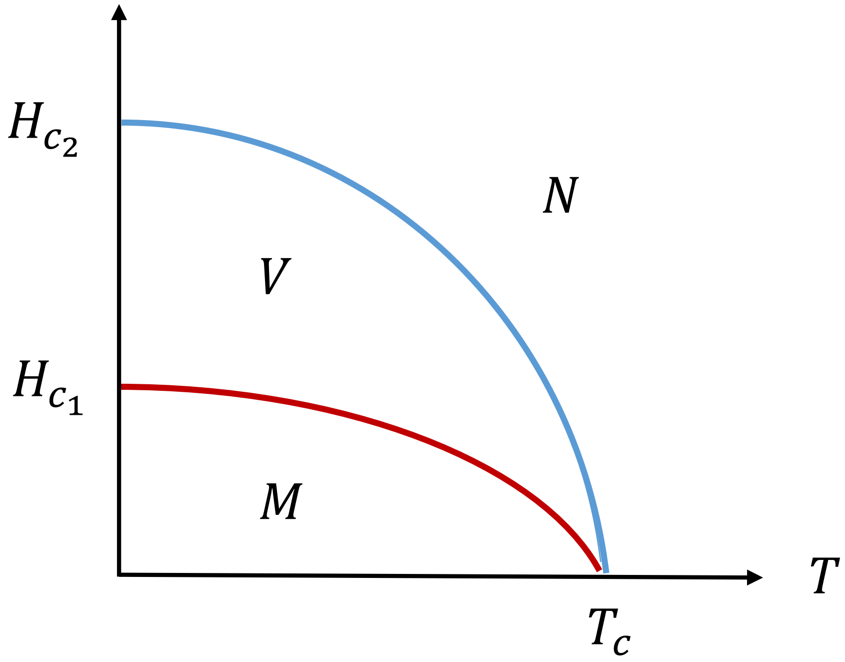
Phenomena:

1. Two critical magnetic fields (Hc_1, Hc_2)



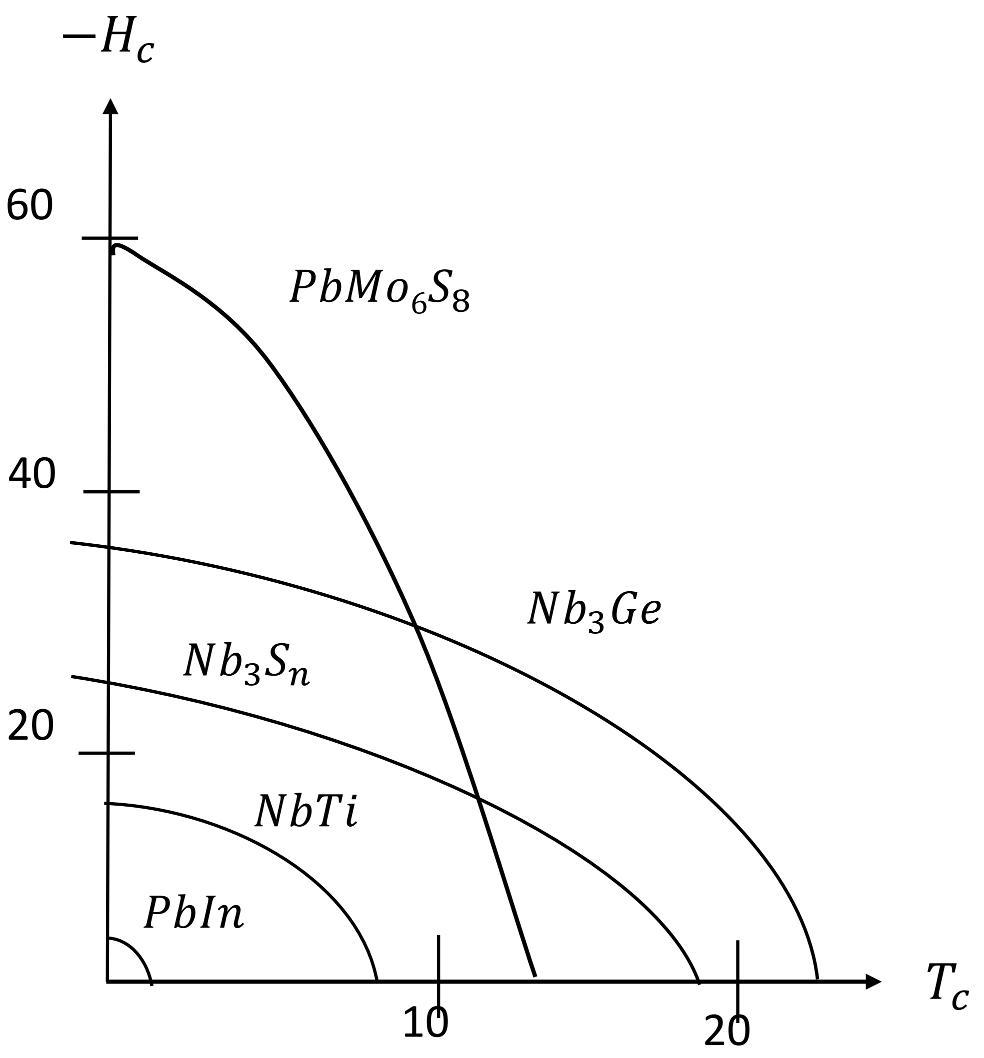
Hc_1 field starts to penetrate

Hc_2 field fully penetrates (normal state)

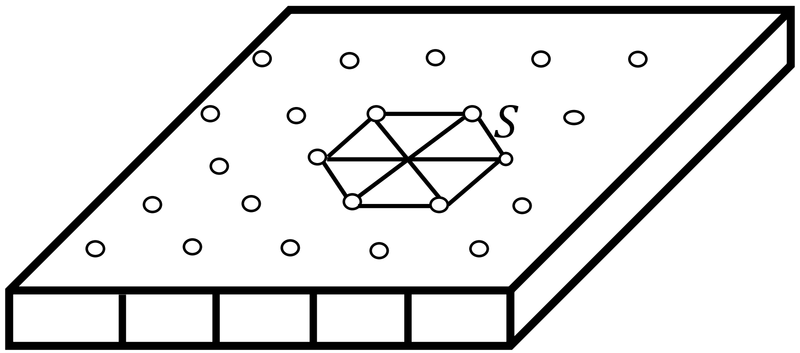


2. H_{c2} field scale can be much larger

T_c		H_{c2}
$6K$	$PbIn$	$0.2\ T$
$10K$	$NbTi$	$13\ T$
$18K$	Nb_3S_n	$23\ T$
$15K$	$PbMo_6S_8$	$60\ T$
$90K$	$YBCO$	$\sim 100\ T$?

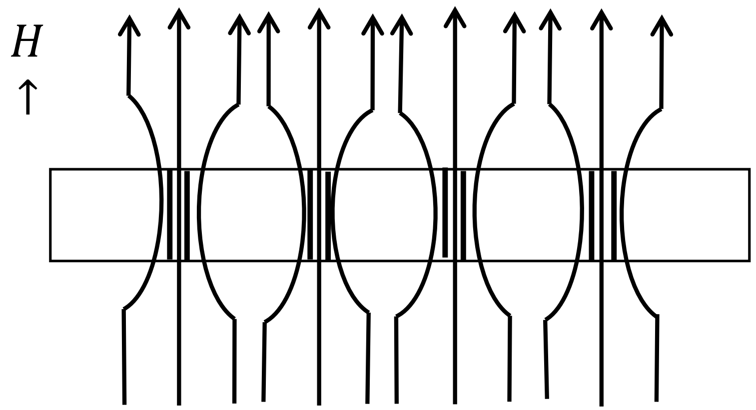


Vortex state ($H_{c_1} < H_2 < H_{c_2}$)



Triangular lattice of vortices of flux Φ_0

$$\Phi_0 = \frac{h}{2e}$$
$$= 2.07 \times 10^{-7}\ G - cm^2$$
$$= 2.07 \times 10^{-15}\ Wb$$



$$n = \frac{B}{\Phi_0}$$

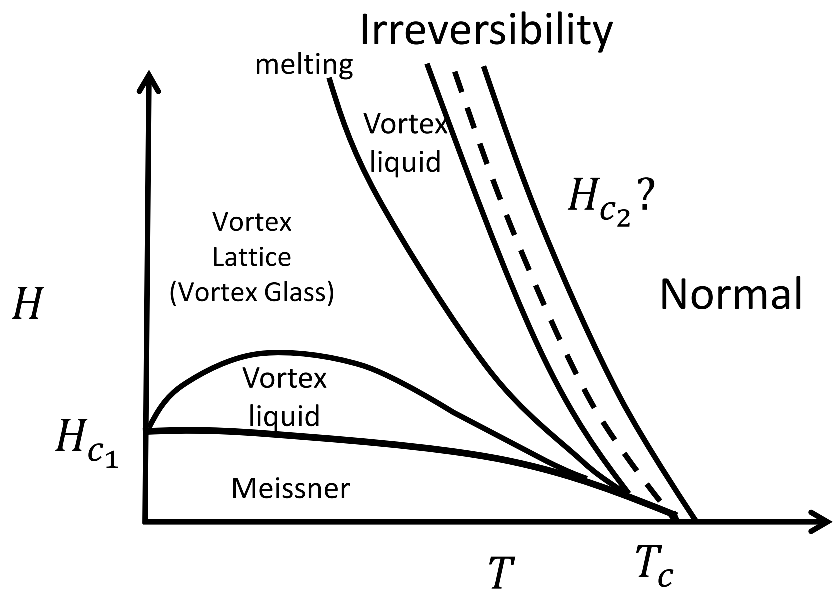
$$s = \left(\frac{2}{\sqrt{3}\,n}\right)^{1/2}$$

Maximize interface area subject to flux quantization constraint

(c) Anisotropic materials

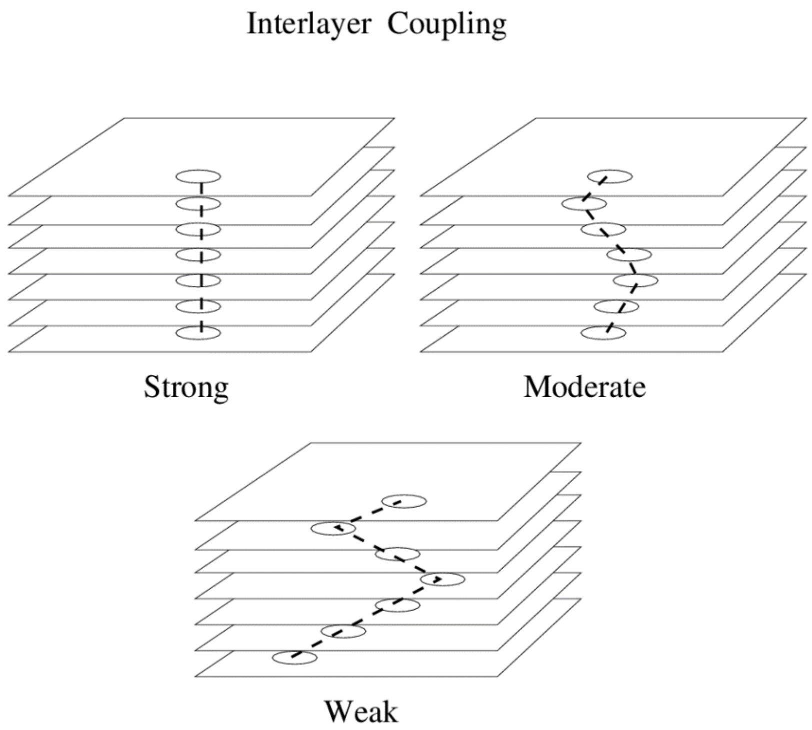
HTSC are very anisotropic --- layered materials with weak interlayer coupling

This allows vortices to decouple between layers, giving rise to a complex vortex phase diagram



Abrikosov vortices in bulk material

Pancake vortices in layered material

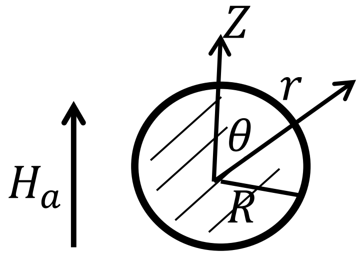


See review by Blatter et al.

(d) Geometric effects --- the intermediate state

Screening of fields is affected by the geometry of the sample $\rightarrow$ demagnetizing effects

Consider a SC sphere:



Inside: $\vec{B} = 0 = \vec{H} + 4\pi\vec{M}$

Outside: $\vec{\nabla} \cdot \vec{B} = 0 \qquad \vec{\nabla} \times \vec{B} = 0$

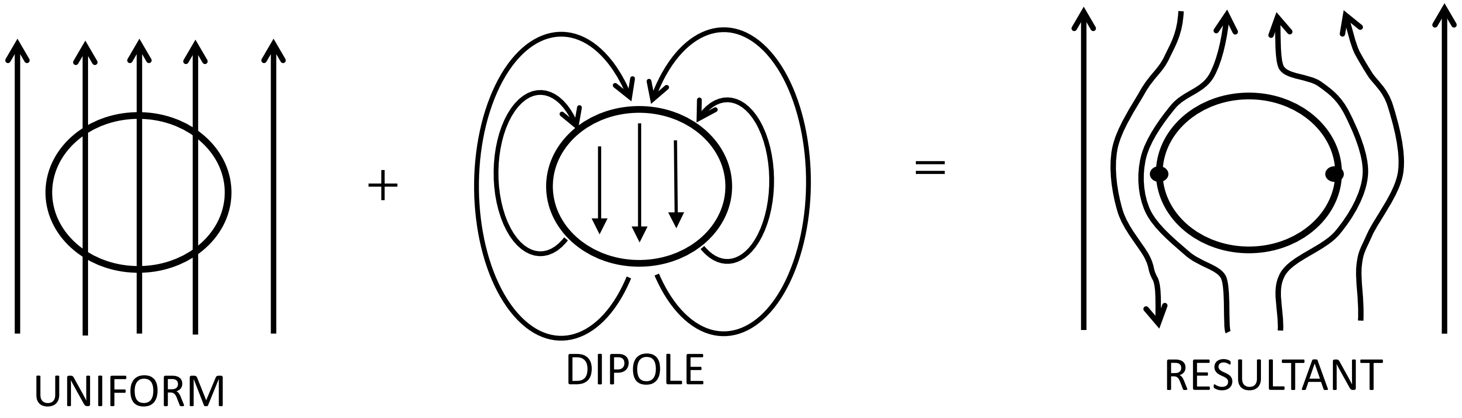
B.C: $B \rightarrow Ha \;\; as \quad r \rightarrow \infty$

$\vec{B} \cdot \hat{n} = 0 \qquad r = R$

H_t continuous at $r = R$

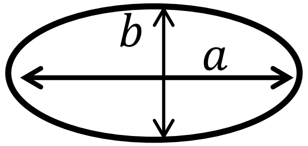
OUTSIDE: $\vec{B} = \vec{H}_a + \frac{H_a R^3}{2}\vec{\nabla}\left(\frac{cos\theta}{r^2}\right) = \vec{H}$

INSIDE: $\vec{B} = 0 \qquad \vec{H} = \frac{3}{2}Ha \qquad \vec{M} = -\frac{3}{8\pi}H_a$

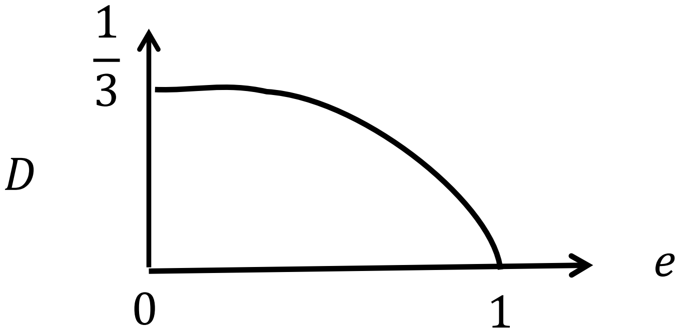


Demagnetizing Factor D

For ellipsoidal sample:



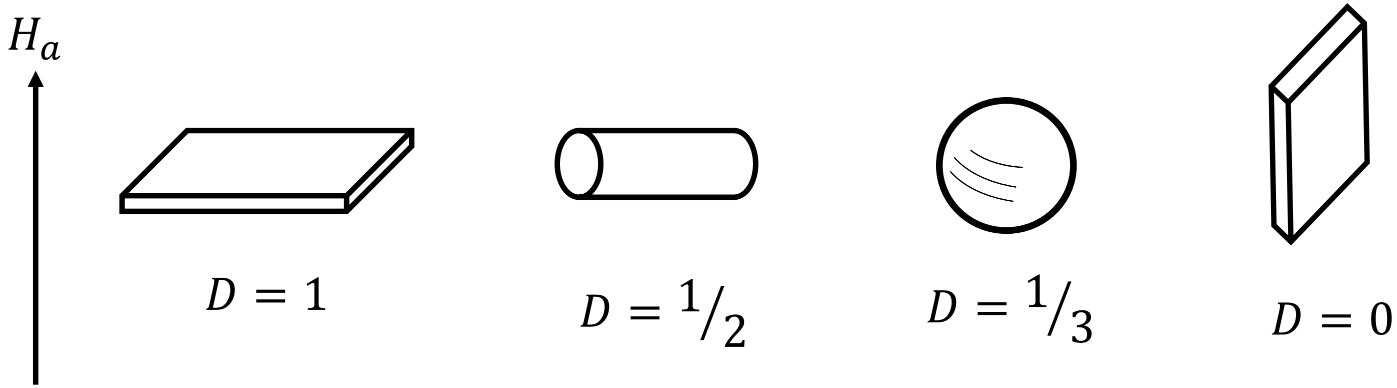
$\rightarrow H_a$



$$D = \left(\frac{1}{e^2} - 1\right)\left[\frac{1}{2e}\ln\left(\frac{1+e}{1-e}\right) - 1\right]$$

where e = $\frac{a-b}{a+b}$ = eccentricity

In general, Define demagnetizing factor for each shape:



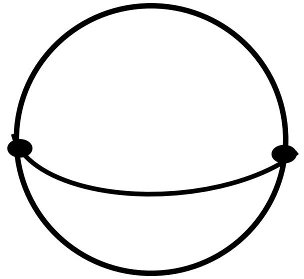
$D = 1$ $\qquad D = {}^{1}/_{2}$ $\qquad D = {}^{1}/_{3}$ $\qquad D = 0$

$\vec{B} = \vec{H} + 4\pi\vec{M}$ $\qquad \vec{H} = H_a - (4\pi M)D$

$\vec{B} = 0$
(inside)

$\vec{H} = \frac{H_a}{(1-D)}$
(inside)

$\vec{M} = -\frac{H_a}{4\pi(1-D)}$
(outside)



Since H_t continuous, $H_{eq} = B_{eq} = H$ inside

Look at equator: $H_{eq} = \frac{H_a}{1-D} > H_a$ for D>0

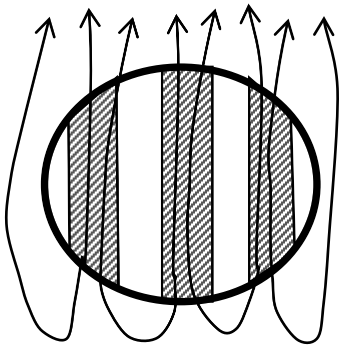
Turn up applied field, $H_{eq} >> H_c$ before $H_a = H_c$

PARADOX: system cannot stay SC $(H > Hc \Rightarrow N)$

system cannot go N (field penetrates so that H = $H_a < H_c \Rightarrow SC$)

edge cannot go N (still same problem)

SOLUTION: system breaks up into N and S longitudinal



INTERMEDIATE STATE (between the Meissner and Normal states)

(like vortex state since flux penetrates but tries to minimize rather than maximize the amount of surface)

- Field range:

- Fields:

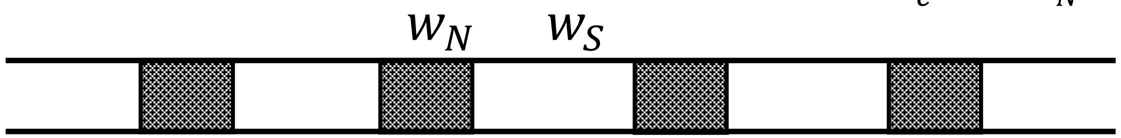
In S: $B = 0 \quad H = H_c \quad M = -\frac{H_c}{4\pi}$

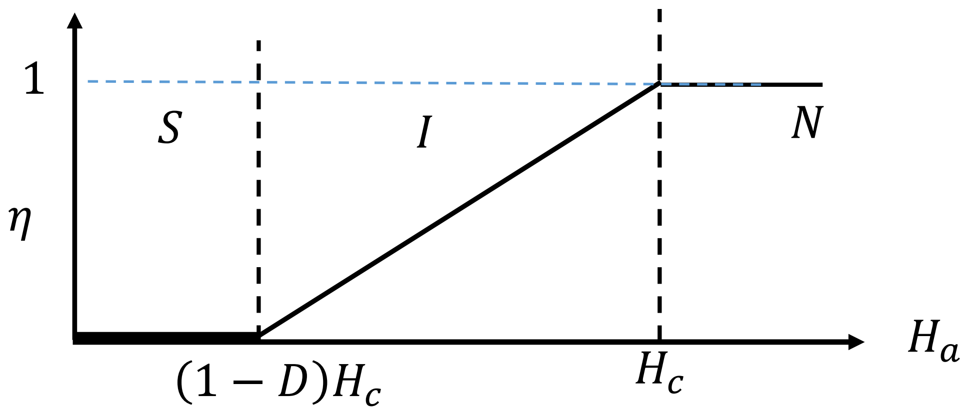
In N: $B = H = H_c \quad M = 0$ (if more or less, interface would move as material goes $N \leftrightarrow S$)

- Fraction of N material ($\equiv \eta$):

Conserve flux through sample to determine η:

$D = 1$: $\langle B \rangle = \eta H_c = H_a \Longrightarrow \eta = \frac{H_a}{H_c} = \frac{w_N}{w_N + w_S}$





$D < 1$: $\langle B \rangle = \eta H_c = [H_a - (1-D)H_c]/D \Longrightarrow \eta = \frac{H_a - (1-D)H_c}{DH_c}$

Derivation of η expression :

Consider sample as having an “average” magnetization <M> <u>less</u> than perfect diamagnetism value:

$$H = Ha - 4\pi < M > D = Hc \quad \Rightarrow \quad < M > = \frac{1}{4\pi} \frac{H_a - Hc}{D}$$

Compare to <M> in laminar structure:

$$< M > = \eta < \cancelto{0}{MN} > + (1-\eta) < MS > = -\tfrac{1}{4\pi}(1-\eta) H_c$$

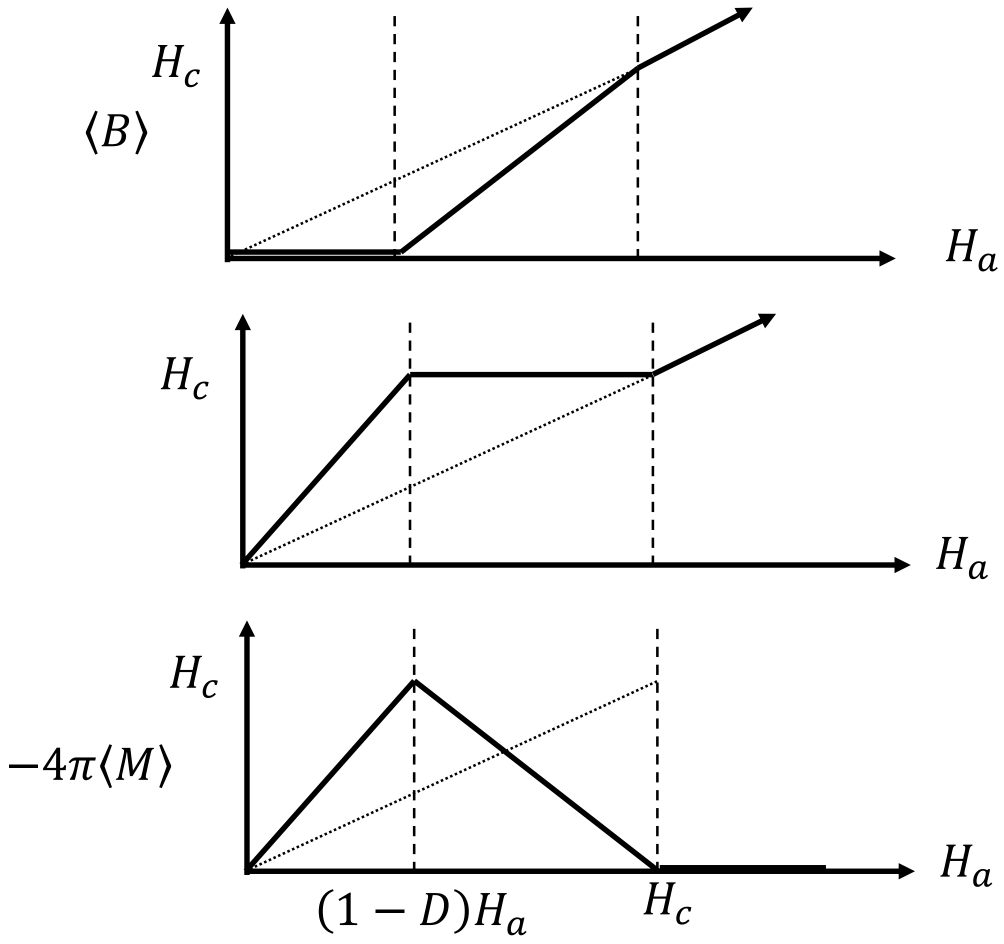
$$(\eta - 1) H_c = \frac{1}{4\pi} \frac{H_a - Hc}{D} \quad \Rightarrow \eta = \boxed{\frac{H_a - (1-D) H_c}{D H_c}}$$

Magnetic Properties (averaged properties)

$$< B > = \underbrace{(1-\eta)0}_{S} + \underbrace{\eta H_c}_{N} = \eta H_c$$

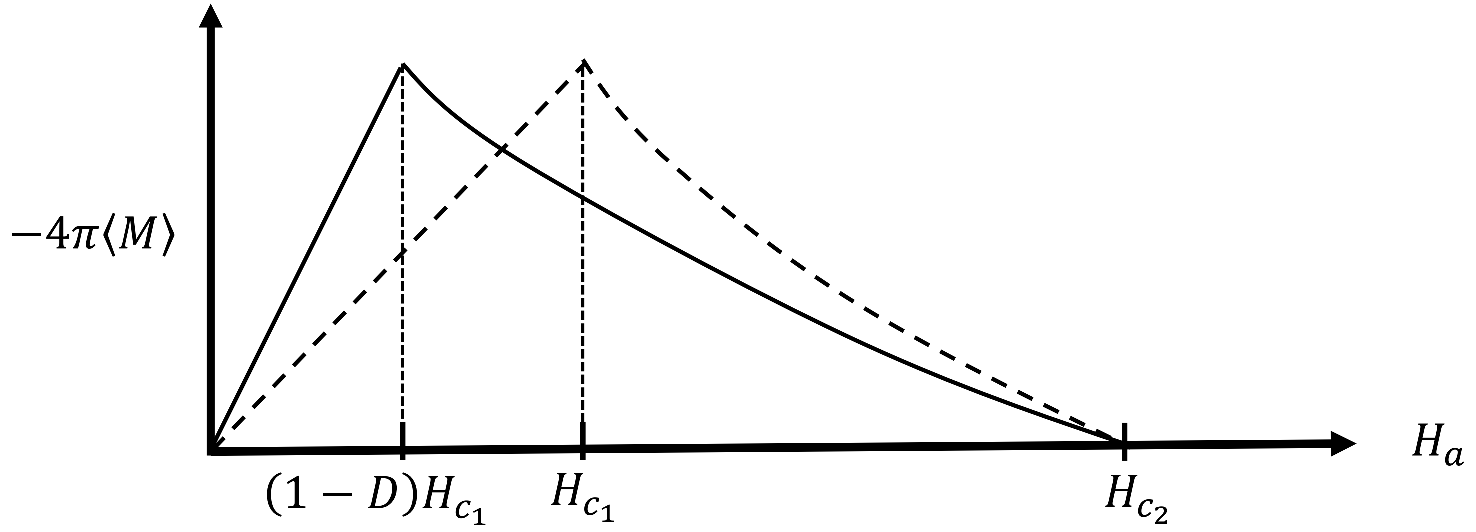
$$< H > = H_c$$

$$< M > = < B >< H > = (\eta - 1)H_c$$



Intermediate state in Type II SCs

Same effects <u>below</u> H_{c1} – distorts magnetization



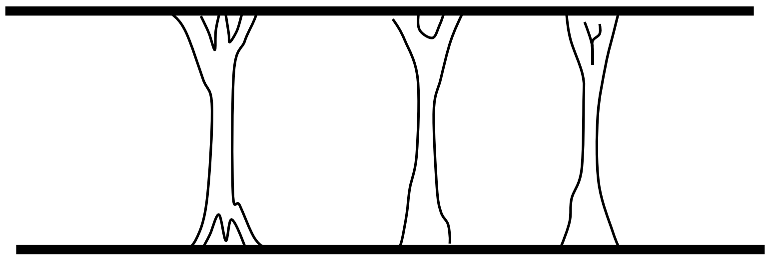
Laminae Structures

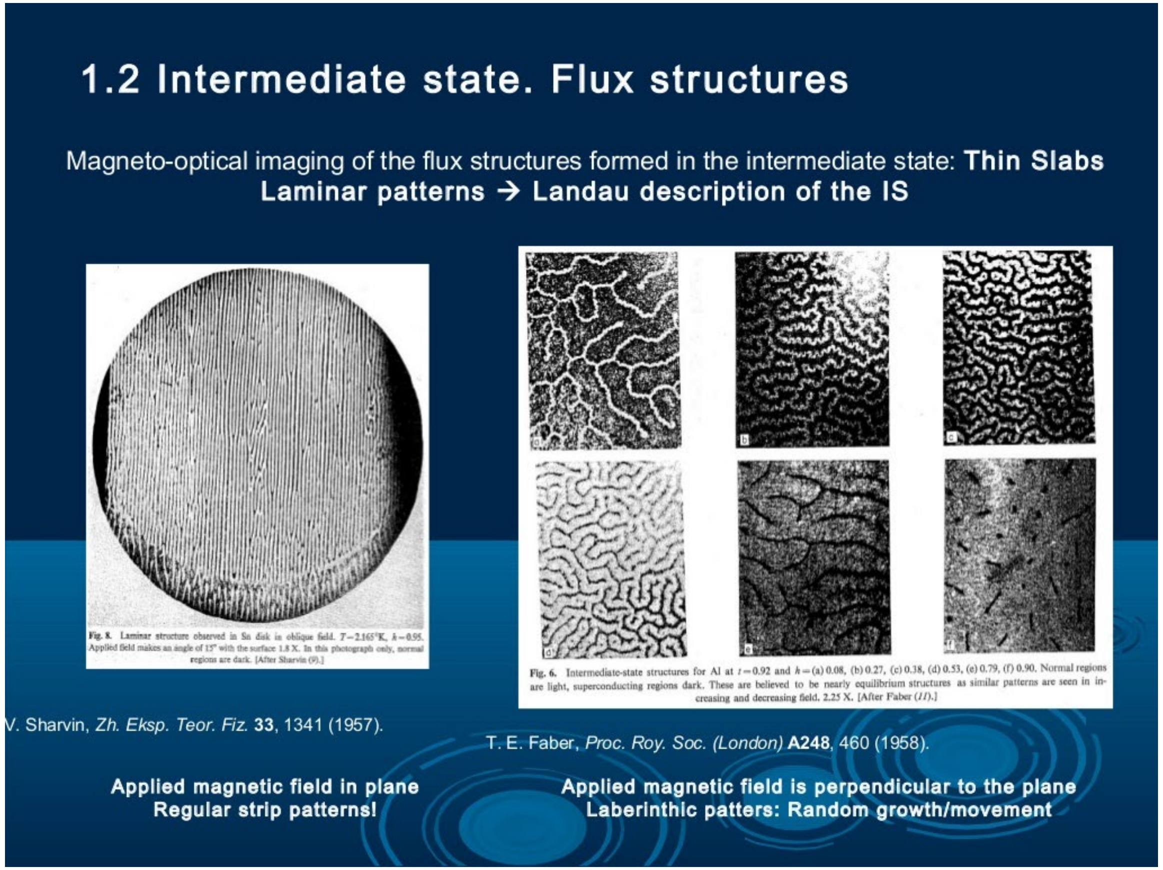
COMPLEX structure and calculation

- Ideal – depends on geometry, fields, boundary energy (N/S)
- Real – also depends on surface condition and sample homogeneity

Best model: Landau Domain Theory

Add energies:
condensation (SC) energy of S
field energy of N
spreading energy of field outside
wall energy (N/S)
branching energy (domain broadening near the surface





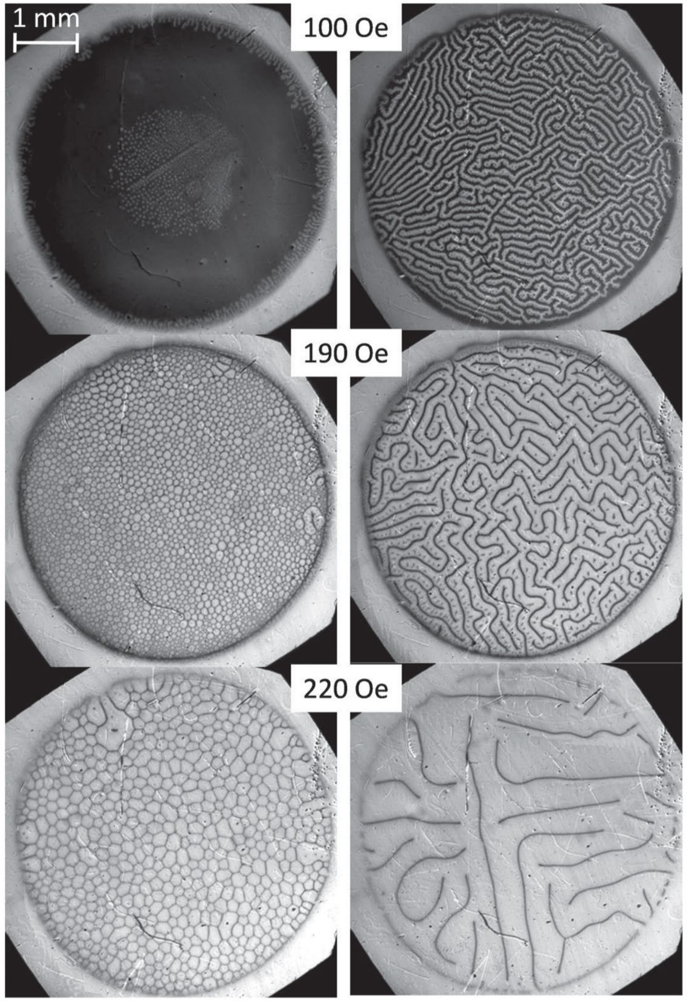
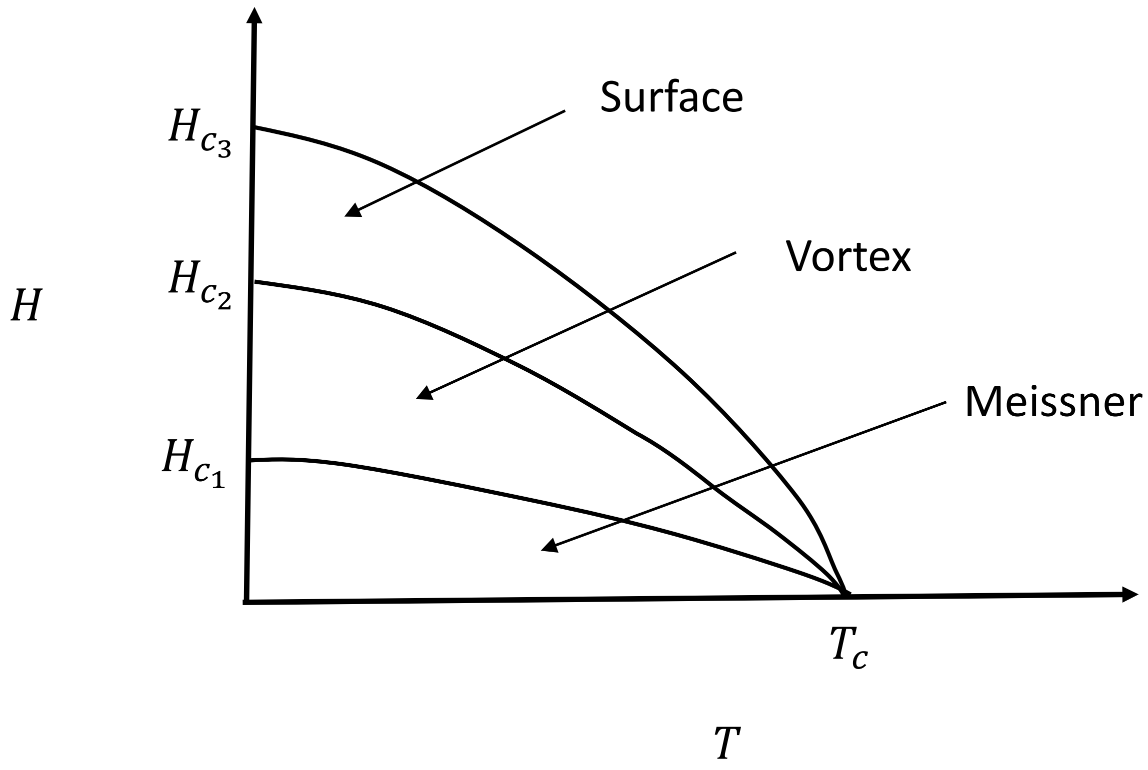


FIG. 2. Structure of the intermediate state in a disc-shaped Pb single crystal at 5 K. Left column—increasing magnetic field after ZFC. Right column—decreasing field.

(e) Surface superconductivity

Nucleation of SC favored near the surface $\Rightarrow$ SC can exist at higher fields at surfaces

$H_{c3} \sim 1.7\, Hc_2$



Physics: SC order parameter is suppressed at the surface (costing energy) but allowing field penetration (saving energy) --- we will see this trade-off in the Ginzburg-Landau model